as their guides. In order to judge the case in question, the analogy of the Thomas Christians is instructive. They were not baptized by an apostle as their own tradition recorded, but the sect originated at a much later time.

The literature on the white and black Jews in Malabar might be considerably multiplied, but without throwing any light on the historicity of the native tradition.

TÜBINGEN, GERMANY. R. GARBE.

MATHEMATICIANS AND PHILOSOPHERS.

Jonathan Swift, in the second chapter of that part of *Gulliver's Travels* which describes Gulliver's third voyage, made Gulliver say of the mathematicians of Laputa:

"They are very bad reasoners and vehemently given to opposition unless when they happen to be of the right opinion, which is seldom their case. Imagination, fancy and invention they are wholly strangers to, nor have they any words in their language by which those ideas can be expressed; the whole compass of their thoughts and mind being shut up within the two fore-mentioned sciences [mathematics and music].

"Most of them, and especially those who deal in the astronomical part, have great faith in judicial astrology although they are ashamed to own it publicly. But what I chiefly admired, and thought altogether unaccountable, was the strong disposition I observed in them towards news and politics, perpetually inquiring into public affairs, giving their judgments in matters of state, and passionately disputing every inch of a party opinion. I have indeed observed the same disposition among most of the mathematicians I have known in Europe, although I could never discover the least analogy between the two sciences."

Gulliver's Travels was published in 1726. In 1734 George Berkeley (1685-1753), the famous philosopher and Bishop of Cloyne in Ireland, published *The Analyst, or a Discourse to an Infidel Mathematician,*[1] the object of which was to show that the principles of the infinitesimal calculus are no clearer than, or perhaps not as clear as, the principles of Christianity. The "infidel mathe-

[1] Cf. on this book and the controversy to which it gave rise M. Cantor, *Vorlesungen über Geschichte der Mathematik*, Vol. III, 2d. ed., Leipsic, 1901, pp. 737-746; Cf. Vol. IV (Leipsic, 1908; article by G. Vivanti, pp. 644, 648, 649) for references to Berkeley on the subject of the theory of the compensation of errors with Lagrange and Lazare Carnot.

matician" referred to was Edmund Halley (1656-1742), the distinguished astronomer and friend of Isaac Newton, and of whom the story is told[2] that, when he indulged in jest concerning theological questions, he was curtly repulsed by Newton with the remark: "I have studied these things; you have not!"

The incident which led Berkeley to write his *Analyst* was this. A friend of Berkeley's refused the offer of spiritual consolation when he was on a bed of sickness because Halley, the skilled mathematician, had convinced him of the inconceivability of the doctrines of Christianity. Now Berkeley showed very skilfully that the principles of infinitesimal analysis—the method of fluxions, in Newton's terminology—were by no means clear. The *Analyst* provoked a great deal of controversy: Dr. James Jurin wrote under a pseudonym *Geometry No Friend to Infidelity*; a Dublin professor named Walton wrote a *Vindication of Sir Isaac Newton's Principles of Fluxions*; Berkeley replied with a *Defence of Freethinking in Mathematics*; Jurin wrote in 1735 *The Minute Mathematician, or the Freethinker no Just Thinker*;[3] Benjamin Robins and Pemberton entered the lists on behalf of Newton's method of fluxions and against Jurin's clumsy defence of it; and lastly, Berkeley's *Analyst* has the credit of inspiring Maclaurin to write his famous *Treatise of Fuxions* of 1742, in which a rigorous foundation of the method of fluxions was attempted.

"The *Analyst*," said De Morgan,* "was intentionally a publication involving the principle of Dr. Whately's argument against the existence of Buonaparte; and Berkeley was strictly to take what he found. The *Analyst* is a tract which could not have been written except by a person who knew how to answer it. But it is singular that Berkeley, though he makes his fictitious character nearly as clear as afterwards did Whately, has generally been treated as a real opponent of fluxions. Let us hope that the arch Archbishop [Whately] will fare better than the arch Bishop."

But Berkeley's tract had another merit. In it are given the foundations of the theory that the correct results of the infinitesimal calculus are obtained by a compensation of errors. This theory

[2] Mach, *Mechanics* (Chicago, 1907), pp. 448-449.

[3] The most comical mis-translation I have ever come across is of the title of a work of which this reminds me, I mean Berkeley's *Alciphron, or the Minute Philosopher* (1733)—a dialogue in which he critically examines the various forms of freethinking in the age. A "minute philosopher" is, of course, a philosopher who examines things minutely; but Montucla, in his *Histoire des Mathématiques*, translates it as "le petit philosophe"!

* *Phil. Mag.*, Nov., 1852.

was rediscovered, apparently independently of others and of each other, by Lagrange and Lazare Carnot in 1797.

In view of this merit, R. Adamson[4] seems rather too severe. Speaking of another work of Berkeley's he says that a "great part of the *Common Place Book* [containing Berkeley's thoughts on physics and philosophy from about 1703] is occupied with a vigorous and in many points exceedingly ignorant polemic against the fundamental conception of the fluxional and infinitesimal calculus, a polemic which Berkeley carried on to the end of his days." Also it is hardly correct to say, with Adamson: ".... in his *Analyst* he attacked the higher mathematics, as leading to freethinking; this involved him in a hot controversy." Berkeley did not attack mathematics; only the vague ideas and expositions of some mathematicians.

If the fact that Berkeley's early mathematical work was bad is any excuse for depreciating his later mathematical work, then indeed the *Analyst* might excusably be condemned; for Berkeley's early mathematical work of 1707 was, according to Cantor,[5] insignificant. But the theory of the compensation of errors in the infinitesimal calculus is perfectly correct and was, apparently,[6] stated for the first time by Berkeley.

* * *

Most mathematicians, like the unphilosophical among men of science, are so occupied with the use of methods which, judged by the results of years of application to the problems of nature, are manifestly reliable, that they too often succumb to the temptation of taking d'Alembert's maxim for mathematicians—*Allez en avant, la foi vous viendra*—as a maxim for logicians and philosophers, and of treating with hasty contempt the criticisms of philosophers. In the case of Berkeley, the mathematicians missed the point as much as Dr. Johnson did when he refuted Berkeleyanism by kicking a stone. In the case of Hegel, the mathematicians appear to have been more in the right. The principal criticism to which Hegel's criticism is subject is that it is too uncritical: it accepts some mathematician's obscurities and concludes obscurity in mathematics. The mathematicians denied obscurity in mathematics, and they were right. But still the fundamental conceptions

[4] *Encycl. Brit.*, 9th ed., Vol. III, 1875, p. 590.

[5] *Op. cit.*, Vol. III, p. 737.

[6] Berkeley's opponents, apparently mistakenly, said that the idea of compensation was old (*ibid.*, pp. 743-744).

of the infinitesimal calculus had not been presented in a logically rigorous form when Hegel wrote.

The care for exactness in dealing with principles is of comparative late growth in mathematics. We shall not be far wrong if we put its birth after Kant published his great *Critique* in 1781. I cannot find any evidence for a direct influence of Kant on Lagrange, Gauss, Cauchy, or Weierstrass; it seems that criticism was "in the air." And so, in the settlement of the logical and philosophical difficulties of mathematics philosophers have not hitherto had a large share. ".... Philosophy asks of Mathematics: What does it mean? Mathematics in the past was unable to answer, and Philosophy answered by introducing the totally irrelevant notion of mind. But now Mathematics is able to answer, so far at least as to reduce the whole of its propositions to certain fundamental notions of logic. At this point, the discussion must be resumed by Philosophy."[7]

* * *

There is another aspect of the distinction between mathematicians and logicians. In modern times, from the time of Leibniz up to the middle of the nineteenth century, the only mathematicians of eminence who were also eminent logicians were John Wallis (1616-1703) and perhaps Leonhard Euler (1707-1783). About the middle of the nineteenth century there began, of course, with Boole and De Morgan, a new era for logic, in which the symbolism and methods of algebra were used to give generality and precision to logical conclusions and to create new logical methods. "Every science," says De Morgan,[8] "that has *thriven* has thriven upon its own symbols: logic, the only science which is admitted to have made no improvements in century after century, is the only one which has *grown no symbols.*" Again, De Morgan in his *Syllabus,*[9] says: "I end with a word on the new symbols which I have employed. Most writers on logic strongly object to all symbols except the venerable *Barbara, Celarent,* etc..... I should advise the reader not to make up his mind on this point until he has well weighed two facts which nobody disputes, both separately and in connection. Firstly, logic is the only science which has made no progress since the revival of letters; secondly, logic is the only science which has produced no growth of symbols."

[7] B. Russell, *The Principles of Mathematics,* Cambridge, 1903, p. 4; cf. pp. 129-130.

[8] *Trans. Camb. Phil. Soc.,* Vol. X, 1864, p. 184.

[9] *Syllabus of a Proposed System of Logic,* London, 1860, p. 72.

But De Morgan saw the advantages that would result from the use by logic of a symbolism analogous to the algebraical. In his third paper "On the Syllogism"[10] he says: "As joint attention to logic and mathematics increases, a logic will grow up among the mathematicians, distinguished from the logic of the logicians by having the mathematical elements properly subordinated to the rest. This 'mathematical logic'—so-called *quasi lucus a non nimis lucendo*—will commend itself to the educated world by showing an actual representation of their form of thought—a representation, the truth of which they recognize—instead of a mutilated and one-sided fragment, founded upon canons of which they neither feel the force nor see the utility."

* * *

At the present time the prejudice of logicians against the use of symbols that happen to have been used beforehand in mathematics has almost disappeared, thanks principally to the work of Dr. J. Venn.[11] The whole objection of the old-fashioned logicians really rested on no better grounds than this: The use of x and y, which are used in mathematics, ought not to be used instead of the logical X and Y, *because x and y have been used for something quantitative.*[12] As if the word "cabbage" had any rigid connection with the essence of the vegetable of which that word reminds us! In symbolic logic the arithmetical signs +, −, and × were used because certain logical operations have many analogies with arithmetical operations. There is no objection that can be urged against this—a fertile source of discoveries—*except* that when we write out mathematical theorems in symbolic logic there may be a confusion of terms. But we must be careful not to pursue the analogy too far. Logical addition, for example, and mathematical addition are not identical. There is a point where the analogy breaks down. And when we go deeply into the matter, the differences will begin to outweigh the identities in importance. Broadly speaking, we may say that *modern logic is symbolic but has got beyond the rather evident analogies it has with algebra,* and a man who seemed rather deep and abstract in his mathematics and logic fifty years ago now seems rather a superficial and *naïf* person. In one form or another, analogy probably is always guiding us in our researches, but, as we

[10] *Trans. Camb. Phil. Soc.,* Vol. X, 1864, note on page 176.

[11] *Symbolic Logic,* London, 1881, 2d ed. 1894.

[12] Cf. *ibid.,* p. ix (of either edition).

progress in subtlety, we become more and more convinced of the limitations of the more obvious analogies.

* * *

Let us now return to Swift. In his description of Gulliver's voyage to Laputa, he describes the mathematicians of that country as silly and useless dreamers, whose attention has to be awakened by flappers. Also, the mathematical tailor measures his height by a quadrant, and deduces his other dimensions by a rule and compasses, producing a suit of very illfitting clothes. On the other hand, the mathematicians of Laputa, by their marvelous invention of the magnetic island floating in the air, ruled the country and maintained their ascendency over their subjects. Dr. Whitehead[13] says: "Swift, indeed, lived at a time peculiarly unsuited for gibes at contemporary mathematicians. Newton's *Principia* had just been written, one of the great forces which have transformed the modern world. Swift might just as well have laughed at an earthquake." We cannot wholly subscribe to this, for it seems not unlikely that Swift, like everybody else, could not doubt the usefulness, importance, and correctnesss of the mathematician's work, but shared, with the philosopher, a doubt of the mathematician's being able to state his principles clearly and reasonably, just as we may doubt the existence of a knowledge of thermodynamics in a man who drives a railway engine.

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CURRENT PERIODICALS.

In the number of *Scientia* for August, 1915, Georges Bohn gives the second part of his article on new ideas on adaptation and evolution. It is interesting to notice that, according to the author, both Lamarck and Darwin were finalists. E. Carnevale contributes the second part of his study on democracy and penal justice. The articles concerned with questions raised by the war are by W. J. Ashley on "The Economic Conversion of England" and Charles Guignebert on the part played by the Roman Catholic Church— or what, according to him is the same thing, the Pope—in the European war. There is a short note by Federigo Enriques on the art of writing a treatise, prompted by his forthcoming book on the geometrical theory of equations and algebraic functions. There are reviews of books and periodicals, and French translations of the Italian and English articles.

[13] A. N. Whitehead, *An Introduction to Mathematics*, London, 1911, p. 10.